Reg. No: | | | | | | | | | |

G .T.N. ARTS COLLEGE (AUTONOMOUS)

(Affiliated to Madurai Kamaraj University)
(Accredited by NAAC with 'B' Grade)

SUMMATIVE EXAMINATION - NOVEMBER 2017

Class : **I M.Sc., Mathematics** Date : **06.11.2017**
Paper Code : **17PMAC11** Time : **10.00 a.m to 01.00 p.m**
Title of the Paper : **Algebra - I** Max Marks : **75**

Section – A **[10 X 1 = 10]**

[Answer ALL the Questions]

1.The number of conjugate classes in S_n is _____

[a] $p(n!)$ [b] $p(n+1)$ [c] $p(n-1)$ [d] $p(n)$.

2. The normalizer $N(a)$ of an element in a group G is ____

[a] $\{x \in G / xa = x\}$ [b] $\{x \in G / xa = a\}$ [c] $\{x \in G / xa = ax\}$ [d] $\{x \in G / xa = e\}$

3. Suppose that G is the internal direct product of $N_1, N_2 \ldots\ldots N_n$. If $a \in N_i$, $b \in N_j$ $(i \neq j)$ then ___.

[a] $ab = e$ [b] $ab = ba$ [c] $ab = a$ [d] $ab = b$

4. The number of non-isomorphic abelian groups of order 2^4 equals

[a] 5 [b] 4 [c] 3 [d] 2

5. The number of ideals of a field is

[a] 1 [b] 2 [c] 3 [d] 4

6. If π is a prime element in the Euclidean ring R and π/ab where a,b ε R Then

[a] π/a [b] π/b [c] [a] and [b] [d] [a] or [b]

7. If a+ib is not a unit of J [i] then

[a] $a^2 + b^2 < 1$ [b] $a^2 + b^2 > 1$ [c] $a^2 + b^2 = 1$ [d] a + b = 1

8. The polynomical $x^2 + 1$ is

[a] irreducible over R [b] monic [c] [a] and [b] [d] [a] or [b]

9. A field is said to be an extension of F if

[a] F contains K [b] K does not contain F [c] K contains F [d] F does not contain K

10. The algebraic numbers forms a

[a] field [b] ring [c] integral domain [d] [a]and[b]

Section – B **[5 X 7 = 35]**

[Answer ALL the Questions]

11 a). If p is a prime number and $p / o(G)$ then prove that G has an element of order P.

[OR]

b). If $o(G)=p^n$ where p is a prime number then prove that $Z(G) \neq (e)$.

12 a). If G and G^1 are isomorphic abelian groups then prove that for every integer s, G(s) and $G^1(s)$ are isomorphic.

[OR]

b). Let G be a group and suppose that G is the internal direct product $N_1, N_2 \ldots N_n$. Let $T = N_1 \times N_2 \ldots \times N_n$. Then prove that G and T are isomorphic.

13 a). Let R be a commutative ring with unit element whose only ideals are (0) and R itself . Then prove that R is a field.

[OR]

b). Let R be a Euclidean ring. Then prove that any two elements $a, b \in R$ have a greatest common d of the form $\lambda a + \mu b$ for some $\lambda, \mu \in R$.

14a). State and prove Fermat's theorem.

[OR]

b). State and prove Einstein criterian.

15 a). If L is an algebraic extension of K and if K an algebraic extension of F then prove that L is an algebraic extension of F.

[OR]

b). Prove that a polynomial of degree n over a field can have atmost 'n' roots in any extension field.

Section – C **[3 X 10 = 30]**

[Answer Any THREE Questions]

16. State and prove third part of sylow's theorem.
17. Prove that every finite abelian group is the direct product of cyclic groups.
18. Prove that every integral domain can be imbedded in a field.
19. Prove that J[i] is a Euclidean ring.
20. Prove that the number **e** is transcendental.

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SUMMATIVE EXAMINATION - NOVEMBER 2017

Class : **I M.Sc., (Mathematics)** Date : **09.11.2017**

Paper Code : **17PMAC12** Time : **10.00 a.m to 01.00 p.m**

Title of the Paper : **Analysis I** Max Marks : **75**

Section – A **[10 X 1 = 10]**

[Answer ALL the Questions]

1. All compact metric space and all Euclidean space are ___________
   [a] Compact [b] Connected [c] Complete [d] Continuous

2. $\lim_{n \to \infty} \left(1+\frac{1}{n}\right)^n =$ _________
   [a] e^n [b] e [c] e^{-1} [d] e^{-n}

3. The series $\sum a_n$ is said to converge absolutely if the series ________converges
   [a] $\sum a_n$ [b] $\sum|a_n|$ [c] $|\sum a_n|$ [d] $|a_n|$

4. If $\sum a_n$ is a series of complex numbers which converges absolutely then every rearrangement of $\sum a_n$ __________
   [a] Diverges [b] Converges [c] Continuous [d] Bounded

5. Every uniformly continuous function is__________
   [a] Converges and Continuous [b] Continuous [c] not continuous [d] not converges

6. A mapping f of a set E into R^k is said to be bounded if there is a real number M such that _______ $\forall$ $x \in$ E
   [a] $|f(x)| \geq M$ [b] $|f(x)| \leq M$ [c] $|f(x)| \neq M$ [d] None of these

7. Monotonic functions have no ________of the second kind
   [a] Continuous [b] Uniformly continuous
   [c] Discontinuities [d] Converges

8. The function $f(x) = \begin{cases} 1 & if \quad x \quad is \quad rational \\ 0 & if \quad x \quad is \quad irrational \end{cases}$ then f ___________
   [a] f has a discontinuity of the second kind at every point x
   [b] f has a continuity of the second kind at every point x
   [c] f has a continuous at x = 0
   [d] f has a continuous at x = 1

9. Let f be defined on [a,b], if f has a local maximum at a point $x \in (a,b)$ and if $f'(x)$ exists then ________
   [a] $f'(x) \neq 0$ [b] $f'(x) = 0$ [c] $f'(x) > 0$ [d] $f'(x) < 0$

10. Let f be defined on [a b].If f is differentiable at a point $x \in [a,b]$ then f is __________at x
    [a] continuous [b] uniformly continuous [c] bounded [d] converges

Section – B **[5 X 7 = 35]**

[Answer ALL the Questions]

11. a) State and Prove Root Test

[OR]

b) If $\{P_n\}$ is a sequence in a compact metric space X then prove that some subsequence of $\{P_n\}$ converges to a point of X

12. a) Suppose (i) the partial sums A_n of $\sum a_n$ form a bounded sequence

(ii) $b_0 \geq b_1 \geq b_2 \geq ....$

(iii) $\lim_{n \to \infty} b_n = 0$

Then prove that $\sum a_n b_n$ converges

[OR]

b) Define Rearrangements with example

13.a) A mapping f of a metric space X into a metric space Y is continuous on X iff $f^{-1}(V)$ is open in X for every open set V in Y

[OR]

b) If f is a continuous mapping of a compact metric space X into a metric space Y then prove that $f(X)$ is compact

14. a) If f is a continuous mapping of a compact metric space X into a metric space Yand E isa connected subset of X then prove that $f(E)$ is connected

[OR]

b) Discuss the discontinuity of the second kind at every point by an example of a real valued function

15. a) State and prove Mean Value Theorem

[OR]

b) Suppose f is a real differentiable function on [a, b]and suppose $f'(a) < \lambda < f'(b)$.Then prove that there is a point $x \in (a,b)$ such that $f'(x) = \lambda$

Section – C **[3 X 10 = 30]**

[Answer Any THREE Questions]

16. State and prove Cauchy criterion for convergence

17. Suppose (i) $\sum_{n=0}^{\infty} a_n$ converges absolutely (ii) $\sum_{n=0}^{\infty} a_n = A$ (iii) $\sum_{n=0}^{\infty} b_n = B$

(iv) $\sum_{k=0}^{n} a_k b_{n-k}$ (n=0,1,2,3,….) Then prove that $\sum_{n=0}^{\infty} c_n = AB$

18. Let E be a non compact set in R .Then Prove that
- (i) There exists a continuous function on E which is not bounded
- (ii) There exists a continuous function and bounded function on E which has no maximum
- (iii) There exists a continuous function on E which is not uniformly continuous

19. Let f be monotonic on (a,b).Then prove that the se of points of (a,b) at which f is discontinuous is at most countable

20. State and prove Taylor's Theorem

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SUMMATIVE EXAMINATION - NOVEMBER 2017

Class : **I M.Sc. Mathematics** Date : **13.11.2017**

Paper Code : **17PMAC13** Time : **10.00 a.m to 01.00 p.m**

Title of the Paper : **ORDINARY DIFFERENTIAL EQUATIONS** Max Marks : **75**

Section – A **[10 X 1 = 10]**

[Answer ALL the Questions]

1. If $y_1,y_2,y_3\ldots y_n$ are linear dependent there exist constants $c_1,c_2,..c_n$ (not all zero) such that $c_1y_1+c_2y_2+\ldots c_ny_n=0$where $c_1=c_2=..c_n=0$ then $y_1,y_2,..y_n$ are said to be……
   [a] Linear dependent [b] linear independent
   [c] linear combination [d] either a or b

2. Find there exist n linear independent solutions of L(y)=…. on I
   [a] 0 [b] 1 [c] 2 [d] (0,1)

3. A linear differential equation of the form ($a_0 x^n D^n + a_1x^{n-1} D^{n-1}+a_{n-1}xD$)y=x where $ba_0,a_1,a_2\ldots a_n$ are constants and x is either a constants is called a….
   [a] Non homogeneous linear differential equation [b] Homogeneous linear differential equation
   [c] Euler linear differential equation [d] Cauchy linear differential equation

4. Find the Complimentary function of (D^2-4D +5)y=0
   [a] $2i\pm1$ [b] $2\pm i$ [c] $1\pm3i$ [d] $3\pm2i$

5. The formula of particular of $\frac{1}{f(D)}x^m =$……. f(m)≠0
   [a] $f(m)x^m$ [b] $x^m\frac{1}{f(m)}$ [c] $\left[f(m)^x\right]$ [d] $\frac{f(m)}{x^m}$

6. The legendre's linear equation is …..
   [a] $\left[a_0(a+bx)^nD^n+\ldots.+a_n\right]y=0$ [b] $\left[a_0(a+bx)^nD^n+\ldots.+a_n\right]y=\mathrm{L(Y)}$
   [c] $\left[a_0(a+bx)^nD^n+\ldots.+a_n\right]y=\mathrm{I}$ [d] $\left[a_0(a+bx)^nD^n+\ldots.+a_n\right]y=\mathrm{x}$

7. Find first successive approximation of the solution of $y'=e^x+y^{2,}$,y(0)=0
   [a] $y_1=e^x-1$ [b] $y_1=e^x$ [c] $y_1=1-e^x$ [d] $y_1=e^{-x}$

8. The nth approximation (y_n, z_n) to the initial value problem is ------
   [a] $y_n = y_0 + \int_{x_0}^{x} f(x,y_{n-1},z_{n-1})dx : Z_n = z_0 + \int_{x_0}^{x} f(x,y_{n-1},z_{n-1})dx$
   [b] $y_n = y_0 - \int_{x_0}^{x} f(x,y_{n-1},z_{n-1})dx : Z_n = z_0 - \int_{x_0}^{x} f(x,y_{n-1},z_{n-1})dx$
   [c] $y_n = y_0 - \int_{x_0}^{x} f(x,y_{n-1},z_{n-1})dx : Z_n = z_0 + \int_{x_0}^{x} f(x,y_{n-1},z_{n-1})dx$
   [d] $y_n = y_0 + \int_{x_0}^{x} f(x,y_{n-1},z_{n-1})dx : Z_n = z_0 - \int_{x_0}^{x} f(x,y_{n-1},z_{n-1})dx$

9. A differential equation of the form (r(x)y')' + [q(x)+ λ p(x)] y =0 is called as...
[a] Euler equation [b] picard equation [c] lipschitz vequation [d] strum liouville equation

10. The eigen functions $y_n(x)$ with the corresponding eigen values λ_n are given by
$y_n(x)=e^x \sin \mu_n^x and \lambda = \mu_n^2, n = 1,2,...where \mu_n$ are positive roots of ….
[a] $\mu = \tan \mu a$ [b] $\mu_n = \tan \mu a$ [c] $\mu_3 = \tan \mu a$ [d] $\mu = \tan \mu^5 a$

Section – B **[5 X 7 = 35]**

[Answer ALL the Questions]

11. a) Show that the solutions e^x, e^{-x}, e^{2x}, of (y''') – 2(y'')-(y') + 2y =0 are φ independents and hence or otherwise solve the given equation.

[OR]

b) If $y_1(x)$ and $y_2(x)$ are any two solutions of $a_0(x)y''(x) + a_1(x)y'(x) + a_2(x)y(x)=0$, then the linear combination $c_1y_1+c_2y_2$ where c_1 and c_2 are constant , is also a solution of the given equation.

12. a) Solve x^3(y''') + $2x^2$(y'') +3x(y') - 3y =0

[OR]

b) Solve the differential equation $x^2y''+2xy'=\log x$

13. a) Solve $\left(x^3D^3 + 2x^2D^2 + 2\right)y = 10x + \frac{10}{x}$

[OR]

b) Solve $(x+a)^2 y'' - 4(x+a) y'+6y =x$

14. a) Using the picard's of successive approximation, find the 3rd approximation of the solution φ the equation $y'= x+y^2$ where y=0 when x=0

[OR]

b) Find three successive approximation of the solution of $y'=e^x + y^{2}$,y(0)=0

15. a) For the initial value problem $y' = y^2 + \cos^2 x$,y(0)=0 determine the interval of existence of its solution given that R is the rectangle containing origin;

$$R: \left\{(x, y): 0 \le x \le a, |y| \le b, a \succ \frac{1}{2}, b \succ 0\right\}$$

[OR]

b) Find the eigen values and eigen function of the strum liouville problem
$x'' + \lambda x = 0$,x'(0)=0,x''(L)=0.

Section – C **[3 X 10 = 30]**

[Answer Any THREE Questions]

16. State prove Abel's formula for general term.

17. Solve $\left(x^2D^2 + xD + 1\right)y = \log x . \sin \log x$

18. Solve $16(x+1)^4 y^4 + 96(x+1)^3 y^3 + 104(x+1)^2 y^2 + 8(x+a) y^1 + y = x^2+4x+3$

19. Find the 3rd approximation of the solution of the equation
$\frac{dy}{dx} = z, \frac{dz}{dx} = x^2 z + x^4 y$ by picard's method y=5 and z=1 when x=0.

20. Find the Eigen values and eigen function of the strum liouville problem
$y'' + \lambda y = 0$, with y(0)+y'(0) and y(1)+y'(1)=0.

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SUMMATIVE EXAMINATION - NOVEMBER 2017

Class : **I M.Sc. Maths** | Date : **15.11.2017**

Paper Code : **17PMAC14** | Time : **10.00 a.m to 01.00 p.m**

Title of the Paper : **NUMERICAL ANALYSIS** | Max Marks : **75**

Section – A **[10 X 1 = 10]**

[Answer ALL the Questions]

1. If $f(x)=(x-\xi)^m g(x)=0$ where $g(x)$ is bounded and $g(\xi)\neq 0$ then ξ is said to be a ____
   [a] Single root [b] double root [c] multiple root [d] no root.
2. The root of the equation $\cos x - xe^x = 0$ is contained in the interval ____
   [a] (0.5,1) [b] (0,1) [c] (1,2) [d] (0,2)
3. A matrix A is said to be a lower triangular if $a_{ij}=0$ for
   [a] $i>j$ [b] $j>1$ [c] $j>i$ [d] $j\geq i$
4. The eigen values of a positive definite matrix are _____
   [a] All negative [b] all positive [c] equal to zero [d] may be positive (or) negative.
5. The truncation error of a Taylor's series of a polynomial of degree n is ___
   [a] $\frac{1}{(n+1)!}(x-x_0)^{n+1}f^{(n+1)}(\xi)$ [b] $\frac{1}{(n)!}(x-x_0)^{n+1}f^{(n+1)}(\xi)$
   [c] $\frac{1}{(n-1)!}(x-x_0)^{n+1}f^{(n+1)}(\xi)$ [d] $\frac{1}{(n+1)!}(x-x_0)^{n+1}f(\xi)$.
6. $E[f(x_i)]=$ _______
   [a] $f(x_i+h)$ [b] $f(x_i-h)$ [c] $f(x_i-h/2)$ [d] $f(x_i+h/2)$
7. Name the rule for the following expression $\int_a^b f(x)dx=((b-a)/2)[f(a)+f(b)]$
   [a] Trapezoidal rule [b] Simpson's 1/3 rule [c] Simpson's 3/8th rule [d] Runge kutta method
8. Simpson's 3/8th rule is obtained by taking n= ____ n Newton's cote's integration method
   [a] 0 [b] 3 [c] 2 [d] 1.
9. If no product of the dependent variable $y(t)$ with itself or any one of its derivatives occurs then the equation is said to be _______
   [a] Nonlinear [b] linear [c] quadratic [d] Homogenous
10. In the second order Runge Kutta method, $K_1=$ ____
    [a] $hf(t_j,u_j)$ [b] $h+f(t_j,u_j)$ [c] $h-f(t_j,u_j)$ [d] $f(t_j,u_j)$

Section – B **[5 X 7 = 35]**

[Answer ALL the Questions]

11. a) Perform five iterations of the bisection method to obtain the smallest positive root of the equation $f(x)=x^3-5x+1=0$.

[OR]

b) Use synthetic division and perform two iterations by Birge-Vieta method to find the smallest positive root of the equation $x^4 - 3x^3 + 3x^2 - 3x + 2 = 0$.

12. a) Solve the equations $x_1 + x_2 + x_3 = 6$, $3x_1 + 3x_2 + 4x_3 = 20$, $2x_1 + x_2 + 3x_3 = 13$ using Gauss Elimination method.

[OR]

b) Find A^{10} when $A = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$.

13. a) Find the unique polynomial of degree 2 or less such that $f(0) = 1$, $f(1) = 3$, $f(3) = 55$, using the Lagrange interpolation.

[OR]

b) Find the values of $f(-0.5)$ and $f(0.5)$ for the following values of $f(x)$ and $f'(x)$.

x	-1	0	1
$f(x)$	1	1	3
$f'(x)$	-5	1	7

Using piecewise cubic hermite interpolation.

14. a) Find the Jacobian matrix for the system of equations $x^2 + y^2 - x = 0$, $x^2 + y^2 - y = 0$ at the point (1,1).

[OR]

b) Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$ using

(i) Composite Trapezoidal rule

(ii) Composite Simpson's rule with 2, 4, and 8 equal sub intervals.

15. a) Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$ with $h = 0.1, 0.2$ using Euler method.

[OR]

b) Determine the first three non-zero terms in the Taylor series for the initial value problem $u' = t^2 + u^2$.

Section – C **[3 X 10 = 30]**

[Answer Any THREE Questions]

16. Find the root of the equation $\cos x - xe^x = 0$ using Secant and Regula Falsi method.

17. Find all the eigen values and eigen vectors of the matrix

$$A=\begin{bmatrix}1 & \sqrt{2} & 2\\ \sqrt{2} & 3 & \sqrt{2}\\ 2 & \sqrt{2} & 1\end{bmatrix}.$$

18. Calculate the differences and obtain the forward and backward difference polynomials for the following data

x	0.1	0.2	0.3	0.4	0.3
$f(x)$	1.4	1.56	1.76	2.00	2.28

Also interpolate at $x = 0.25, 0.35$.

19. Evaluate the integral $I = \int_1^2\int_1^2 \frac{dx}{1+x}$ using the trapezoidal rule with $h=k=0.5$ *and* $h=k=0.25$.

20. Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$ with $h = 0.2$ on the interval [0,1] using the Fourth order classical Runge Kutta method.

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SUMMATIVE EXAMINATION - NOVEMBER 2017

Class : **I M.Sc. Maths**
Paper Code : **17PMAE11**
Title of the Paper : **INTEGRAL EQUATIONS**

Date : **17.11.2017**
Time : **10.00 a.m to 01.00 p.m**
Max Marks : **75**

Section – A **[10 X 1 = 10]**
[Answer ALL the Questions]

1. Which is the third kind linear integral equation ?

   [a] f(x)g(x)=$\int_a^x K(x,t)y(t)dt.$ [b] g(x)y(x)=f(x)+λ$\int_a^x K(x,t)y(t)dt.$

   [c] $\int_a^b K(x,t)g(t)dt = f(x)$ [d] y(x)=f(x)+λ$\int_a^x K(x,t)y(t)dt.$

2. Which is minkowski inequality
   [a] ||f+g||>||+f||+||g|| [b] ||f+g||<||+f||+||g|| [c] ||f+g||≤||+f||+||g|| [d] ||f+g||=||+f||+||g||

3. If $d^2y|dx^2$ (XY)=0, $Y(a)=Y_1$,$Y(b)=Y_2$ is a _______
   [a] Initial value problem [b] bounded [c] final value problem [d] Boundary value problem

4. The ordinary linear differential equations order n:

   [a] $y^n+a_1(x)y^{n-1}+a_2(X)y^{n-2}+\ldots.+a_n(x)y=\varphi(x)$ [b] $y^n-a_1(x)y^{n-1}-a_2(X)y^{n-2}-\ldots.-a_n(x)y=\varphi(x)$

   [c] $y^n-a_1(x)y^{n+1}+a_2(X)y^{n+2}+\ldots.+a_n(x)y=\varphi(x)$ [d] $y^n+a_1(x)y^{n-1}+a_2(X)y^{n-2}+\ldots.+a_n(x)y\neq\varphi(x)$

5. A homogeneous fredholm integral equation of the 2^{nd} kind is ______

   [a] X(Y)=λ$\int_a^b K(x,t)y(t)dt$ [b] Y(x)=λ$\int_a^b K(x,t)y(t)dt$

   [c] Y(x)=λ$\int_a^b K(t,x)y(x)dt$ [d] Y(x)=λ$\int_a^b K(x)y(t)dt$

6. If φ (x)is continuous and $\varphi(x)\neq 0$ on the interval (a,b) & $\varphi(x) = \lambda_0\int_a^b K(x,t)\varphi(t)dt$.

   Then φ (x) is known as _______
   [a] Eigen value [b] Eigen vectors [c] Eigen function [d] Eigen elements

7. The eigen functions of the homogeneous system is _______
   [a] $(1-\lambda A^T)c=0$ [b] $(1-\lambda A^T)c\neq 0$ [c] $(1-\lambda A^T)c=3$ [d] $(1-\lambda A^T)c=1/2$

8. Any solution $z_0(x)$ of the transposed homogeneous integral equation z(x) corresponding to the eigen value λ_0 is of the form $z_0(x)$ = ______

   [a] $\sum_{0=1}^{r} b_i z_{oi}(x)$ [b] $\sum_{0=1}^{r} z_{oi}(x)$ [c] $\sum_{0=1}^{\pi} b_i z_{oi}(x)$ [d] $\sum_{0=\infty}^{r} b_i z_{oi}(x)$

9. The equation $y_n(x)=f(X)+\sum_{m=1}^{n}\int_a^b K_m(x,t)f(t)dt$ proceeding to the limit as n→∞, we get ____ series.

   [a] continuous [b] Neumann [c] finite [d] none

10. If $k_n(x,t)$ be iterated kernals then $R(x,t\lambda)=\sum_{m=1}^{\infty}\lambda^{m-1}k_{m(x,t)}$ ⍰= ______

[a] $\sqrt{x,t;\,\lambda}$ [b] √x,y [c] √y,z [d] $\sqrt{x,t;\,f}$

Section – B **[5 X 7 = 35]**

[Answer ALL the Questions]

11. a) Show that the function $y(x)=(1+x^2)^{-3/2}$ is a solution of the volterra

integral equation $y(x)=\frac{1}{1+x^2} - \int_0^x t/1+x^2\, y(t)$

[OR]

b) Define fredholm integral equation and Explain any two special cases.

12. a) Convert the following Differential Equation into integral equation y''+y=0 when y(0)=0 =y'(a).

[OR]

b) The integral equation $y(x)=\int_o^x (x-t)y(t)dt - x\int_0^1 (1-t)y(t)dt$ is equivalent to y''-y=0,y(0)=0=y(1)

13. a) Write about characteristic functions(or eigen functions)

[OR]

b) Solve the homogeneous fredholm equation $y(x)=\lambda\int_0^1 e^x e^t\, y(t)dt$

14. a) Solve $y(x)=\cos x+\lambda\int_0^{\pi} \sin x\;\; y(t)dt$

[OR]

b) Solve $y(x)=f(x)+\lambda\int_0^1 xt\;\; y(t)dt$.

15. a) Evaluate the resolvent kernel for what values of λ the solution does not exist. Obtain solution of the integral equation is $y(x)=1+\lambda\int_0^1 (1-3xt)y(t)dt$.

b) Solve the inhomogeneous fredholm integral equation of the 2nd kind $y(x)=2x+\lambda\int_0^1 (x+t)y(t)dt$.

by the method of successive application to the first order taking $y_0(x)=1$.

Section – C **[3 X 10 = 30]**

[Answer Any THREE Questions]

16. Show that function $y(X)=\sin\frac{\pi x}{2}$ is a solution of the fredholm integral equation

$y(x)-\pi 2/4\int_0^1 k(x,t)y(t)dt = \frac{x}{2}$ where the kernel k(x,t) is of the form

$$K(x,t)=\begin{cases}\frac{x(2-t)}{2} 0 \le x \le t\\ \frac{t(2-x)}{2}, t \le x \le 1\end{cases}$$

17. Reduce the following boundary value problem into an integral equation

$y''+\lambda y=0$ with y(0)=y(l)=0.

18. Find the eigen values and eigen function of the homogeneous integral

equation $y(x)= \lambda \int_0^{\pi} (\cos^2 x \cos 2t + \cos 3x \cos^3 t) y(t) dt.$

19. Invert the integral equation $y(x)=f(x)+ \lambda \int_o^{2\pi} (\sin x \cos t) y(t) dt$.

20. Let $y(x)=f(x)+ \lambda \int_a^x k(x,t) y(t) dt$ .be given volterra integral equation of the second kind suppose that

i. Kernel $k(x,t) \neq 0$, is real and continuous R, for which $a \leq x \leq b, a \leq t \leq b$.Also let $|k(x,t)| \leq M$ in R
ii. $f(x) \neq 0$ is real and continuous in the interval I, for which $a \leq x \leq b$. Also let $|f(x)| \leq N$ in I
iii. λ is a contant then I has a unique continuous solution in I and this solution is given by the absolutely and uniformly converges series

$$Y(x)=f(x)+ \lambda \int_a^x k(x,t) y(t) dt + \lambda^2 \int_a^x k(x,t) \int_a k(t,t_1)\, dt_1, dt + \ldots\ldots$$